# 3 Theoretical Background

## 3.1 Kinematic Systems

Kinematic systems (kinematic chains) consist of rigid bodies (links) which are interconnected by joints (kinematic pairs) or constraint elements. These networks can have either a plain tree configuration or contain additional loops (see Fig. 3–1 a. and b.). Furthermore, a special category of chains with loops is represented by the closed chains where every link is coupled to at least two other links. These closed kinematic chains can be either mobile mechanisms or non-mobile static structures. The mechanisms can in general contain lower and higher kinematic pairs while the linkages represent a special class of mechanisms which contain lower kinematic pairs only [1,3].

In some cases a kinematic network can be divided into two or more *Clusters*, sections of a network which are connected to each other by constraint elements only (see Fig. 3–1 c.).

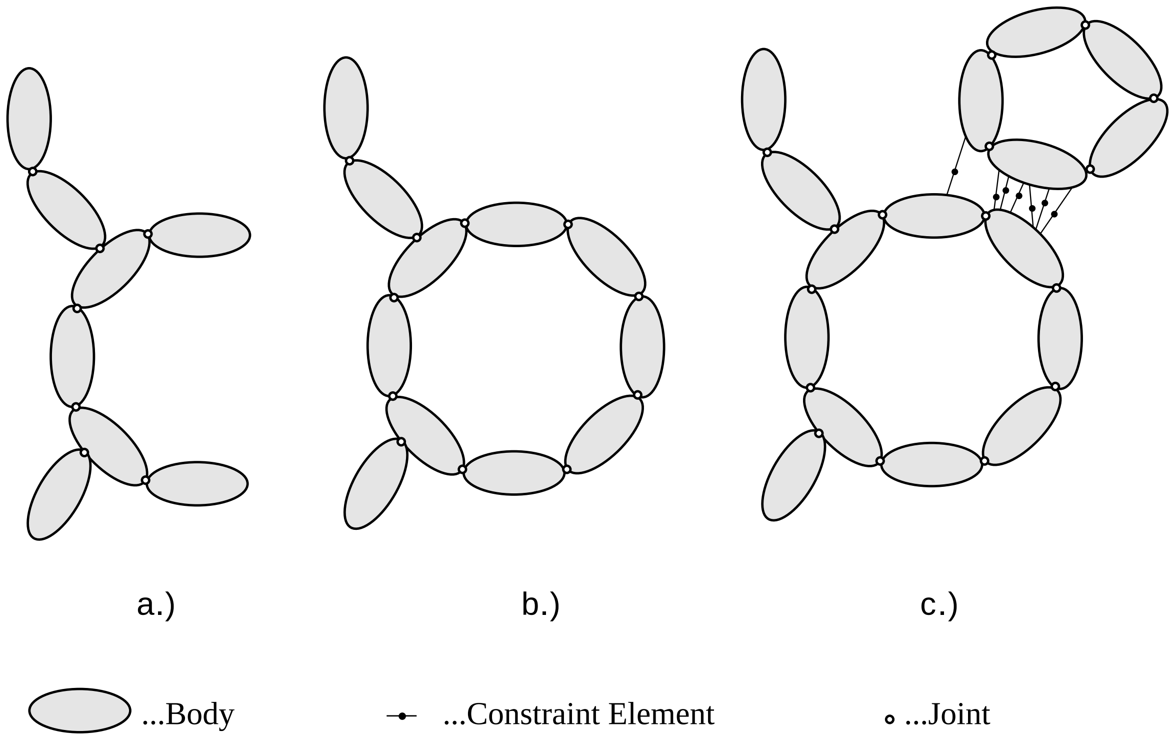


**Fig 3–1:** Types of kinematic systems.

Moreover, the behavior of a kinematic system can be also influenced by mechanical or electronic control devices which is typical for various robot manipulators etc.

## 3.2 Elements of Kinematic Systems

### 3.2.1 Rigid Body

A rigid body is represented by its coordinate system $\underline{\mathbf{e}}^{(i)}$, *body coordinate system* (see Fig. 3–2), whose coordinate axes are defined by the three orthogonal unit vectors $\mathbf{e}_1^{(i)}, \mathbf{e}_2^{(i)}$ and $\mathbf{e}_3^{(i)}$.
Its position and orientation relative to the reference coordinate system $\underline{\mathbf{e}}^{(0)}$ is given by the following homogeneous transformation matrix [4]:

$$\underline{G}_i = \begin{pmatrix} {}^{(0)}_{1}e_1^{(i)} & {}^{(0)}_{1}e_2^{(i)} & {}^{(0)}_{1}e_3^{(i)} & {}^{(0)}_{1}g_i \\ {}^{(0)}_{2}e_1^{(i)} & {}^{(0)}_{2}e_2^{(i)} & {}^{(0)}_{2}e_3^{(i)} & {}^{(0)}_{2}g_i \\ {}^{(0)}_{3}e_1^{(i)} & {}^{(0)}_{3}e_2^{(i)} & {}^{(0)}_{3}e_3^{(i)} & {}^{(0)}_{3}g_i \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} {}^{(0)}\underline{E}^{(i)} & {}^{(0)}\underline{g}_i \\ \underline{0}^{T} & 1 \end{pmatrix} \qquad 3\text{–}1$$

In the matrix $\underline{G}_i$, each element ${}^{(0)}_{m}e_n^{(i)}$ denotes the m-th coordinate of the n-th unit vector $\mathbf{e}_n^{(i)}$ relative to the *reference coordinate system* $\underline{\mathbf{e}}^{(0)}$. In addition, ${}^{(0)}_{m}g_i$ represents the m-th coordinate of the displacement vector $\mathbf{g}_i$ which gives the position of the origin of the i-th *body coordinate system* $\underline{\mathbf{e}}^{(i)}$ relative to the $\underline{\mathbf{e}}^{(0)}$.

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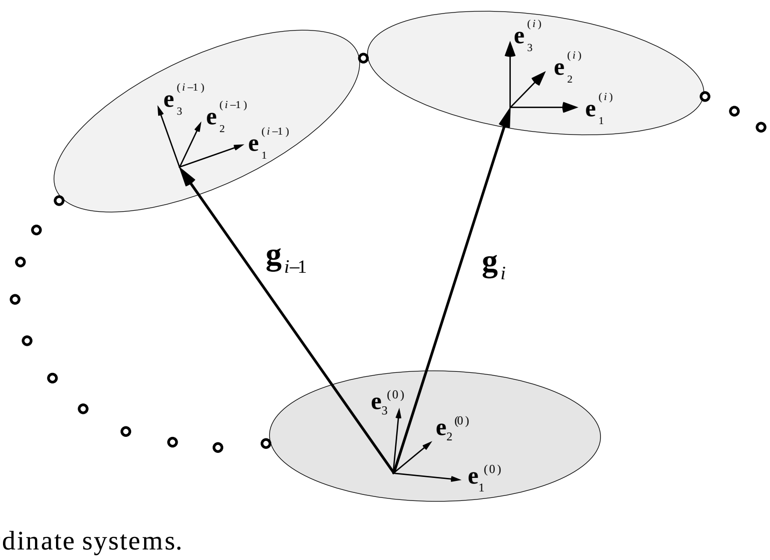


**Fig 3–2:** Body coordinate systems.

### 3.2.2 Joints (Kinematic Pairs)

A joint in general connects two bodies and prescribes their relative motion. The joints can be classified into the lower and higher pairs. Characteristic for the lower pairs is a contact along a surface while in the case of higher pairs a contact takes place along a line or a point [1]. Moreover, independently of the type of a joint, the prescribed relative motion can be given by the homogeneous matrix [4]

$$\underline{\mathrm{R}}_i = \begin{pmatrix} {}^{(i-1)}_{1}e_1^{(i)} & {}^{(i-1)}_{1}e_2^{(i)} & {}^{(i-1)}_{1}e_3^{(i)} & {}^{(i-1)}_{1}r_i \\ {}^{(i-1)}_{2}e_1^{(i)} & {}^{(i-1)}_{2}e_2^{(i)} & {}^{(i-1)}_{2}e_3^{(i)} & {}^{(i-1)}_{2}r_i \\ {}^{(i-1)}_{3}e_1^{(i)} & {}^{(i-1)}_{3}e_2^{(i)} & {}^{(i-1)}_{3}e_3^{(i)} & {}^{(i-1)}_{3}r_i \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} {}^{(i-1)}\underline{\mathrm{E}}^{(i)} & {}^{(i-1)}\underline{\mathrm{r}}_i \\ \underline{0}^{\mathrm{T}} & 1 \end{pmatrix} \qquad 3\text{–}2$$

In the matrix $\underline{\mathrm{R}}_i$, each element ${}^{(i-1)}_{m}e_n^{(i)}$ denotes the m-th coordinate of the n-th unit vector $\mathbf{e}_n^{(i)}$ relative to the foregoing body with the coordinate system $\underline{\mathbf{e}}^{(i-1)}$. The element ${}^{(i-1)}_{m}r_i$ represents the m-th coordinate of the relative displacement vector $\mathbf{r}_i$ which gives the position of the origin of the i-th *body coordinate system* $\underline{\mathbf{e}}^{(i)}$ relative to the neighboring coordinate system $\underline{\mathbf{e}}^{(i-1)}$. The relative transformation $\underline{\mathrm{R}}_i$ is defined as a matrix product

$$\underline{\mathrm{R}}_i = \underline{\mathrm{A}}_{i-1}.\underline{\mathrm{T}}_i.\underline{\mathrm{A}}_i^{-1}. \qquad 3\text{–}3$$

While the form of the matrix $\underline{\mathrm{T}}_i$ depends on the joint type, the matrix $\underline{\mathrm{A}}_i$ is for all joints determined as

$$\underline{\mathrm{A}}_i = \begin{pmatrix} {}^{(i)}_{1}m_i & {}^{(i)}_{1}n_i & {}^{(i)}_{1}(\mathbf{m}_i \times \mathbf{n}_i) & {}^{(i)}_{1}a_i \\ {}^{(i)}_{2}m_i & {}^{(i)}_{2}n_i & {}^{(i)}_{2}(\mathbf{m}_i \times \mathbf{n}_i) & {}^{(i)}_{2}a_i \\ {}^{(i)}_{3}m_i & {}^{(i)}_{3}n_i & {}^{(i)}_{3}(\mathbf{m}_i \times \mathbf{n}_i) & {}^{(i)}_{3}a_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In the matrix $\underline{\mathrm{A}}_i$ the element ${}^{(i)}_{k}m_i$ denotes the k-th coordinate of the vector $\mathbf{m}_i$, ${}^{(i)}_{k}n_i$ represents the k-th coordinate of the vector $\mathbf{n}_i$, and ${}^{(i)}_{k}a_i$ denotes the k-th coordinate of the vector $\mathbf{a}_i$ [4]. All vector coordinates in this matrix are defined relative to the coordinate system in the i-th body. Moreover, the unit vectors $\mathbf{m}_i$, $\mathbf{n}_i$ and $\mathbf{m}_i \times \mathbf{n}_i$ are orthogonal and form a new coordinate system $\underline{\mathbf{a}}^{(i)}$ which is fixed to the i-th body. The vector $\mathbf{a}_i$, on the other hand, determines the distance between the origins of the coordinate systems $\underline{\mathbf{a}}^{(i)}$ and $\underline{\mathbf{e}}^{(i)}$. The coordinate system $\underline{\mathbf{a}}^{(i)}$ is used to determine the position of a joint in the i-th body and the initial relative position and orientation of the connected bodies which is given by

$$\underline{\mathrm{R}}_i = \underline{\mathrm{A}}_{i-1}.\underline{\mathrm{A}}_i^{-1}. \qquad 3\text{–}4$$

The types of joints used in the presented work can be described as follows:

**Revolute Joint**

A revolute joint (R) restricts the relative motion of two connected bodies to a rotation about an axis which is aligned with the vectors $\mathbf{n}_{i-1}$ and $\mathbf{n}_i$ (see Fig. 3–3). Moreover, the origins of the coordinate systems $\underline{\mathbf{a}}^{(i-1)}$ and $\underline{\mathbf{a}}^{(i)}$ coincide in the point $C$, whose position is relative to the coordinate systems $\underline{\mathbf{e}}^{(i-1)}$ and $\underline{\mathbf{e}}^{(i)}$ given by the vectors $\mathbf{a}_{i-1}$ and $\mathbf{a}_i$, respectively. Thus the transformation matrix

$$\underline{\mathrm{T}}_i = \begin{pmatrix} \cos(\theta_i) & 0 & \sin(\theta_i) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_i) & 0 & \cos(\theta_i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

rotates the coordinate system $\underline{\mathbf{a}}^{(i)}$ relative to $\underline{\mathbf{a}}^{(i-1)}$ about the axis which is given by the vectors $\mathbf{n}_{i-1}$ and $\mathbf{n}_i$. The angle of rotation $\theta_i$, which is measured between the unit vectors $\mathbf{m}_{i-1}$ and $\mathbf{m}_i$, is the only variable parameter of a revolute joint and therefore the number of degrees of freedom equals one.

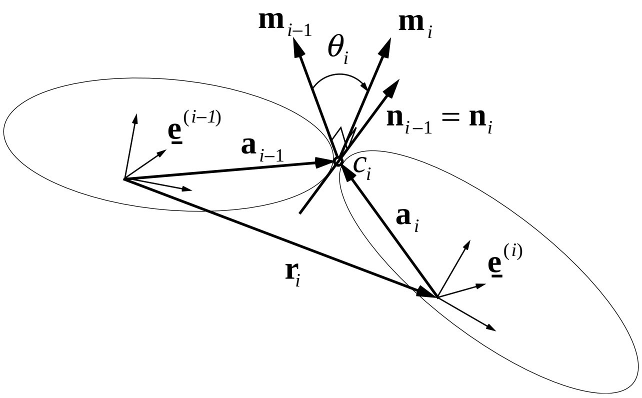


**Fig 3–3:** Revolute Joint

### Prismatic Joint

A prismatic joint (P) restricts the relative motion of two connected bodies to a translation along an axis of translation which is aligned with the vectors $\mathbf{n}_{i-1}$ and $\mathbf{n}_i$ (see Fig. 3–4).

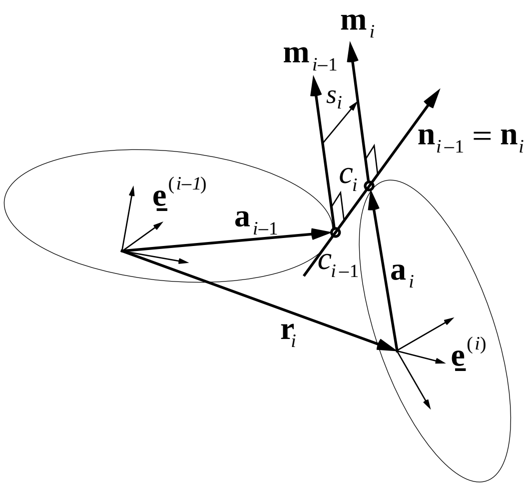


**Fig 3–4:** Prismatic Joint

The translation $s_i$ is the only variable parameter of a prismatic joint and, like in the case of a revolute joint, the number of degrees of freedom equals one.The translation is measured between the points $C_{i-1}$ and $C_i$ which coincide with the origins of the coordinate systems $\underline{\mathbf{a}}^{(i-1)}$ and $\underline{\mathbf{a}}^{(i)}$, respectively, and slide along the axis of translation. The vector $\mathbf{a}_{i-1}$ determines the position of the point $C_{i-1}$ in the coordinate system $\underline{\mathbf{e}}^{(i-1)}$ while the position of the point $C_i$ in $\underline{\mathbf{e}}^{(i)}$ is given by the vector $\mathbf{a}_i$.

The matrix

$$\underline{\mathrm{T}}_i = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

thus translates the coordinate system $\underline{\mathbf{a}}^{(i)}$ relative to $\underline{\mathbf{a}}^{(i-1)}$ along the axis of translation.

### Cylindrical Joint

A cylindrical joint (C) prescribes a relative translation and rotation of two neighboring bodies (see Fig. 3–5).

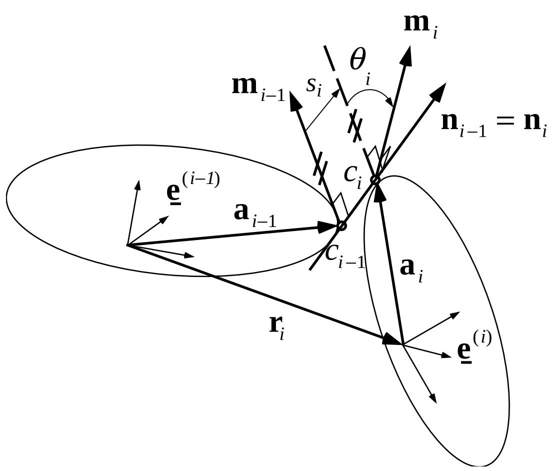


**Fig 3–5:** Cylindrical Joint

It can be viewed as a combination of the prismatic and revolute joints and because of two variable parameters, the rotation $\theta_i$ and translation $s_i$, its number of degrees of freedom equals two. The translation is, like in the case of a prismatic joint, measured between the points $C_{i-1}$ and $C_i$ and the additional rotation about the axis of translation/rotation is measured between the vectors $\mathbf{m}_{i-1}$ and $\mathbf{m}_i$.

The matrix

$$\underline{\mathrm{T}}_i = \begin{pmatrix} \cos(\theta_i) & 0 & \sin(\theta_i) & 0 \\ 0 & 1 & 0 & s_i \\ -\sin(\theta_i) & 0 & \cos(\theta_i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

rotates and translates the coordinate system $\underline{\mathbf{a}}^{(i)}$ relative to the coordinate system $\underline{\mathbf{a}}^{(i-1)}$.

### Helical Joint

Like a cylindrical joint, a helical joint (H) prescribes a relative translation and rotation of two bodies which, in contrast to the cylindrical joint, has only one degree of freedom, introduced by the angle of rotation $\theta_i$. The translation is a function of the rotation angle defined as $s_i = p_i\theta_i$, where $p_i$ denotes the pitch of the helix. The remaining geometrical parameters and the matrix $\underline{\mathrm{T}}_i$ are defined like in the case of a cylindrical joint.

### Global Joint

A global joint (G) restricts the relative motion of two bodies to an arbitrary rotation about the point $C_i$ in which the origins of both coordinate systems $\underline{\mathbf{a}}^{(i-1)}$ and $\underline{\mathbf{a}}^{(i)}$ coincide(see Fig. 3–6). The rotation between the coordinate systems $\underline{\mathbf{a}}^{(i-1)}$ and $\underline{\mathbf{a}}^{(i)}$ is determined by the rotation vector $\mathbf{q}_i$, whose magnitude represents the angle of rotation and the unit vector $\mathbf{u}_i = \mathbf{q}_i / |\mathbf{q}_i|$ denotes the direction of the axis of rotation. The three components of the rotation vector $\mathbf{q}_i$ represent the variable parameters and thus the three degrees of freedom of a global joint. The position of the common point $C_i$ in the coordinate systems $\underline{\mathbf{e}}^{(i-1)}$ and $\underline{\mathbf{e}}^{(i)}$ is given by the vectors $\mathbf{a}_{i-1}$, and $\mathbf{a}_i$, respectively.

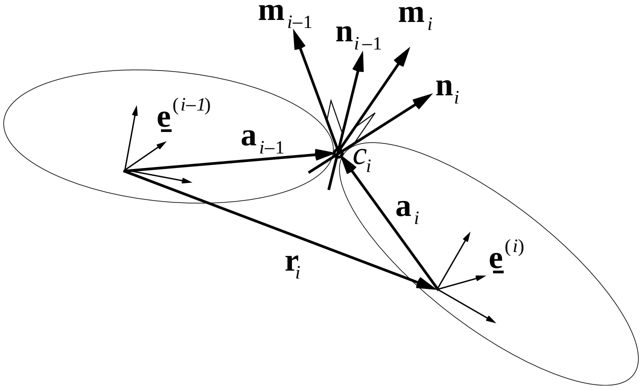


**Fig 3–6:** Global Joint

A global joint can also be interpreted as a combination of three revolute joints with nonparallel axes of rotation. The corresponding matrix $\underline{\mathrm{T}}_i$ rotates the coordinate system $\underline{\mathbf{a}}^{(i)}$ relative to $\underline{\mathbf{a}}^{(i-1)}$. It is defined as [4]

$$\underline{\mathrm{T}}_i = \begin{pmatrix} u_1 u_1 v(\theta)+\cos(\theta) & u_1 u_2 v(\theta)-u_3\sin(\theta) & u_1 u_3 v(\theta)+u_2\sin(\theta) & 0 \\ u_1 u_2 v(\theta)+u_3\sin(\theta) & u_2 u_2 v(\theta)+\cos(\theta) & u_2 u_3 v(\theta)-u_1\sin(\theta) & 0 \\ u_1 u_3 v(\theta)-u_2\sin(\theta) & u_2 u_3 v(\theta)+u_1\sin(\theta) & u_3 u_3 v(\theta)+\cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where $\theta=|\mathbf{q}_i|$, $v(\theta)=1-\cos(\theta)$ and $u_1,u_2,u_3$ are the coordinates of the column matrix ${}^{(i-1)}\underline{\mathrm{u}}_i$ representing the unit vector $\mathbf{u}_i$ in the coordinate system $\underline{\mathbf{e}}^{(i-1)}$.

### User Defined Joint

A "User Defined Joint" allows the simulation of an arbitrary relative motion of two connected bodies (see Fig. 3–7). In this case the coordinate systems $\underline{\mathbf{a}}^{(i-1)}$ and $\underline{\mathbf{a}}^{(i)}$ coincide with the coordinate systems $\underline{\mathbf{e}}^{(i-1)}$ and $\underline{\mathbf{e}}^{(i)}$, respectively. Therefore, the matrices $\underline{\mathrm{A}}_{i-1}$ and $\underline{\mathrm{A}}_i$ equal the unit matrix and the relative transformation is determined as $\underline{\mathrm{R}}_i=\underline{\mathrm{T}}_i$.

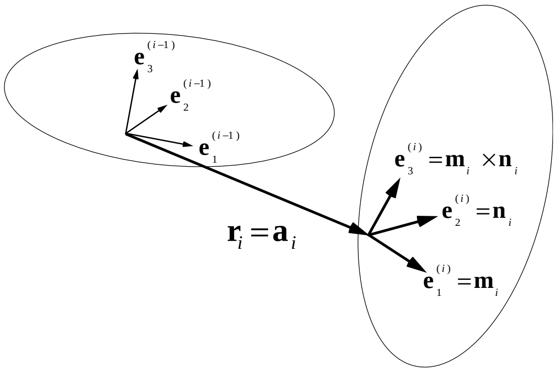


**Fig 3–7:** User Defined Joint

The rotation and translation of the coordinate system $\underline{\mathbf{e}}^{(i)}$ relative to $\underline{\mathbf{e}}^{(i-1)}$ is thus given by the matrix

$$\underline{\mathrm{R}}_i=\underline{\mathrm{T}}_i=\begin{pmatrix} {}^{(i-1)}_{\ \ 1}m_i & {}^{(i-1)}_{\ \ 1}n_i & {}^{(i-1)}_{\ \ 1}(\mathbf{m}_i\times\mathbf{n}_i) & {}^{(i-1)}_{\ \ 1}a_i \\ {}^{(i-1)}_{\ \ 2}m_i & {}^{(i-1)}_{\ \ 2}n_i & {}^{(i-1)}_{\ \ 2}(\mathbf{m}_i\times\mathbf{n}_i) & {}^{(i-1)}_{\ \ 2}a_i \\ {}^{(i-1)}_{\ \ 3}m_i & {}^{(i-1)}_{\ \ 3}n_i & {}^{(i-1)}_{\ \ 3}(\mathbf{m}_i\times\mathbf{n}_i) & {}^{(i-1)}_{\ \ 3}a_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In the matrix $\underline{\mathrm{T}}_i$ the elements ${}^{(i-1)}_{\ \ k}m_i$, ${}^{(i-1)}_{\ \ k}n_i$ and ${}^{(i-1)}_{\ \ k}a_i$ denote the k-th coordinate of the vectors $\mathbf{m}_i$, $\mathbf{n}_i$, and $\mathbf{a}_i$, respectively. The unit vectors $\mathbf{m}_i$, $\mathbf{n}_i$ and $\mathbf{m}_i\times\mathbf{n}_i$ are orthogonal and coincide with the coordinate axes of $\underline{\mathbf{e}}^{(i)}$. The coordinates of these unit vectors thus determine the orientation of the coordinate system $\underline{\mathbf{e}}^{(i)}$ with respect to the $\underline{\mathbf{e}}^{(i-1)}$. The vector $\mathbf{a}_i$, on the other hand, determines the distance between the origins of the coordinate systems $\underline{\mathbf{e}}^{(i-1)}$ and $\underline{\mathbf{e}}^{(i)}$. All vector coordinates in the matrix $\underline{\mathrm{T}}_i$ can be defined by arbitrary functions in such a way that the vectors $\mathbf{m}_i$ and $\mathbf{n}_i$ are always orthogonal unit vectors.

### 3.2.3 Constraint Elements

Constraint elements in general restrict the relative motion of two bodies to some space without prescribing their relative position. Therefore, they can be used in the case where the exact information on the relative position is of no interest. The presented program supports the following constraint elements:

**Global-Global Constraint Element**

A global-global constraint element (G-G), see Fig. 3–8, prescribes the distance $d_i$ between the points $C_{i-1}$ and $C_i$, each belonging to one of the two connected bodies. The position of the point $C_{i-1}$ in the coordinate system $\underline{\mathbf{e}}^{(i-1)}$ is defined by the vector $\mathbf{a}_{i-1}$, and the position of the point $C_i$ in $\underline{\mathbf{e}}^{(i)}$ is defined by the vector $\mathbf{a}_i$. The global-global constraint element can be viewed as a rod which is connected to a body by a global joint at each of its ends.

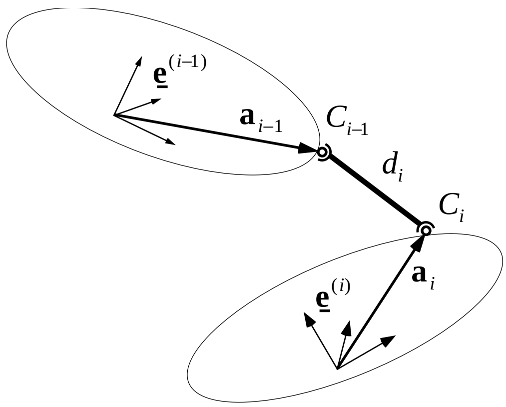


**Fig 3–8:** Global-Global Constraint Element

**Eben-Global Constraint Element**

An eben-global (E-G) constraint element, see Fig. 3–9, restricts the possible motion of the point $C_i$, which is fixed in the coordinate system $\underline{\mathbf{e}}^{(i)}$, to the plane $\vartheta_{i-1}$ which is in turn attached to the coordinate system $\underline{\mathbf{e}}^{(i-1)}$.

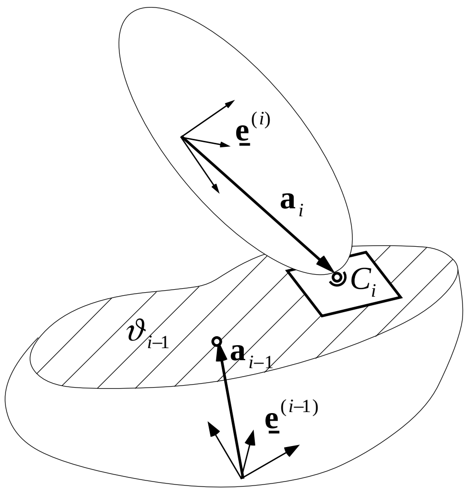


**Fig 3–9:** Eben-Global Constraint Element

The position and orientation of the plane $\vartheta_{i-1}$ in the coordinate system $\underline{\mathbf{e}}^{(i-1)}$ is defined by the vector $\mathbf{a}_{i-1}$ which is normal to the plane. The magnitude of the vector $\mathbf{a}_{i-1}$ represents the shortest distance between the plane and the origin of $\underline{\mathbf{e}}^{(i-1)}$, while the direction of the vector $\mathbf{a}_{i-1}$ defines the orientation of the plane relative to $\underline{\mathbf{e}}^{(i-1)}$. The position of the point $C_i$ in the coordinate system $\underline{\mathbf{e}}^{(i)}$ is defined by the vector $\mathbf{a}_i$.

**Cylindrical-Revolute-Cylindrical Constraint Element**

The C-R-C constraint element determines a pair of skew straight lines $l_{i-1}$ and $l_i$ each representing the axis of rotation and translation of one of the connected bodies (see Fig. 3–10). The axes $l_{i-1}$ and $l_i$ are fixed to the coordinate systems $\underline{\mathbf{e}}^{(i-1)}$ and $\underline{\mathbf{e}}^{(i)}$, respectively, and can be moved relative to each other. Each of these axes can be moved parallel to the other axis and rotated about an axis of rotation which is perpendicular to both of them.

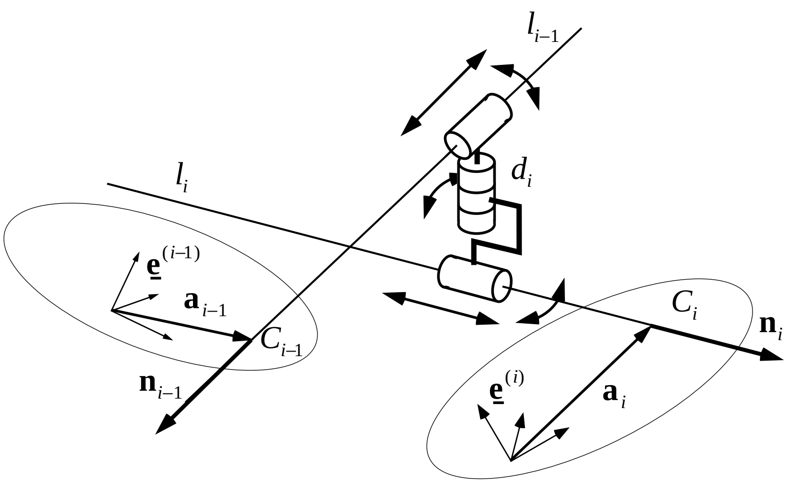


**Fig 3–10:** C-R-C constraint element.

The position of the reference point $C_i$ on the line $l_i$ is determined by the vector $\mathbf{a}_i$ relative to the origin of the coordinate system $\underline{\mathbf{e}}^{(i)}$ and the vector $\mathbf{n}_i$ determines the direction of the line $l_i$. The scalar $d_i$, on the other hand, represents the shortest distance between the lines $l_{i-1}$ and $l_i$.

#### Global-Cylindrical Constraint Element

A global-cylindrical (G-C) constraint element, see Fig. 3–11, restricts the motion of the point $C_i$, which is fixed in the coordinate system $\underline{\mathbf{e}}^{(i)}$, to a cylindrical surface attached to the coordinate system $\underline{\mathbf{e}}^{(i-1)}$. The direction of the symmetry axis of the cylindrical surface is defined by the unit direction vector $\mathbf{n}_i$ and the radius of the cylinder is given by the scalar $d_i$. The global-cylindrical constraint element can be viewed as a rod which is connected to one body by a global joint and to the other body by a cylindrical joint. The rod is perpendicular to the axis of the cylindrical joint.

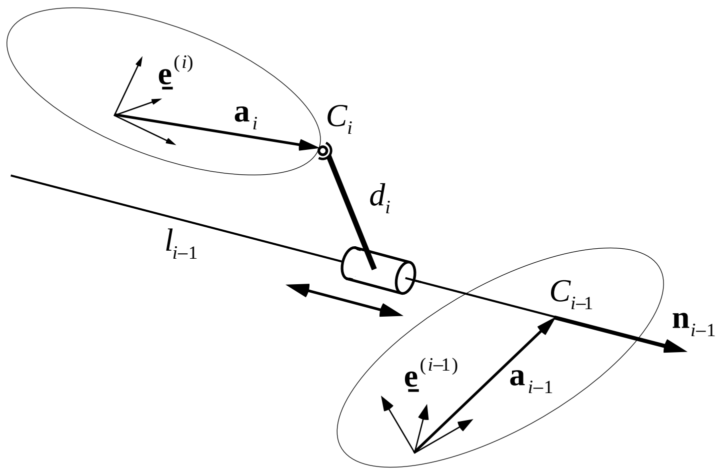


**Fig 3–11:** G-C constraint element.

### 3.2.4 Control Elements

A control element is not a structural element. It merely supplies a common variable parameter which is used in various functions defining the behavior of one or more joints. The motion of two or more joints can be coupled if in each joint at least one or more of its variable parameters are replaced by parametric functions which use at least one common parameter supplied by a control element. Additionally, like in the case of joints, also the lengths of the constraint elements can be influenced by control elements.

For example, having a single-loop mechanism which contains nine revolute joints whose axes of rotation are not parallel to each other, we can define the rotation of three arbitrary joints by different functions with a common variable parameter. In this way, the variable parameter of these functions, supplied by the control element, has replaced three variable parameters of the joints and the total number of variable parameters has been reduced from nine to only seven. In practice, the functions used in these three joints would be realized by electronic or mechanical control devices.

Furthermore, the user defined joint is reasonable only in combination with one or more control elements, supplying the parameters of the functions which define the relative motion of the bodies connected by that joint.

## 3.2.5 Denavit-Hartenberg Parameters (DH Parameters)

The configuration of various simple single-loop mechanisms and opened chains with binary links, such as common robot manipulators, can be in some cases also described by special *joint-body elements*. Each *joint-body element* consists of a joint and a body whose coordinate system $\underline{\mathbf{e}}^{(i)}$ is defined in such a way that the relative position and rotation of the *body coordinate system* $\underline{\mathbf{e}}^{(i+1)}$ of the following *joint-body element* can be determined by the four Denavit-Hartenberg parameters $a_i$, $\alpha_i$, $s_i$ and $\theta_i$. The translation of the *body coordinate system* $\underline{\mathbf{e}}^{(i+1)}$ relative to the $\underline{\mathbf{e}}^{(i)}$ is given by the vector $\mathbf{r}_i$ with coordinates

$$^{(i)}\underline{\mathrm{r}}_i = (s_i\cos(\alpha_i), a_i, -s_i\sin(\alpha_i))^{\mathrm{T}}. \qquad 3\text{–}5$$

Furthermore, its relative rotation is determined by the orthogonal vectors $\mathbf{e}_1^{(i+1)}$ and $\mathbf{e}_2^{(i+1)}$ with the coordinates

$$^{(i)}\underline{\mathrm{e}}_1^{(i+1)} = (\cos(\alpha_i), 0, -\sin(\alpha_i))^{\mathrm{T}} \text{ and} \qquad 3\text{–}6$$

$$^{(i)}\underline{\mathrm{e}}_2^{(i+1)} = (\sin(\alpha_i)\sin(\theta_i), \cos(\theta_i), \cos(\alpha_i)\sin(\theta_i))^{\mathrm{T}}\text{, respectively} \qquad 3\text{–}7$$

In this way the following *joint-body elements* can be defined:

- *Revolute joint-body element*

The relative transformation between the coordinate systems $\underline{\mathbf{e}}^{(i)}$ and $\underline{\mathbf{e}}^{(i+1)}$ is in this case defined by the constant parameters $a_i$, $\alpha_i$, $s_i$ and the variable parameter $\theta_i$.

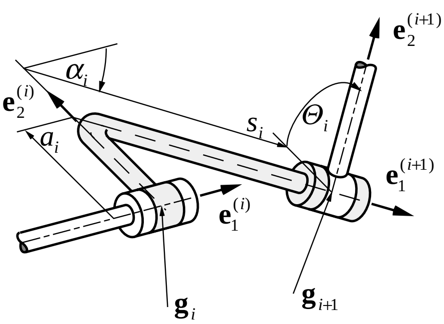


**Fig 3–12:** Revolute joint-body element.

- *Prismatic joint-body element*

The relative transformation between the coordinate systems $\underline{\mathbf{e}}^{(i)}$ and $\underline{\mathbf{e}}^{(i+1)}$ is defined by the constant parameters $a_i$, $\alpha_i$, $\theta_i$ and the variable parameter $s_i$.

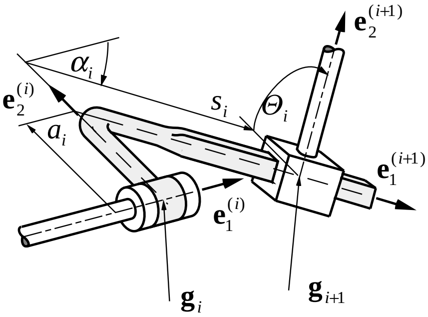


**Fig 3–13:** Prismatic joint-body element.

- *Cylindrical joint-body element*

In this case, the relative transformation between the coordinate systems $\underline{\mathbf{e}}^{(i)}$ and $\underline{\mathbf{e}}^{(i+1)}$ is defined by the constant parameters $a_i$, $\alpha_i$ and the variable parameters $s_i$ and $\theta_i$.

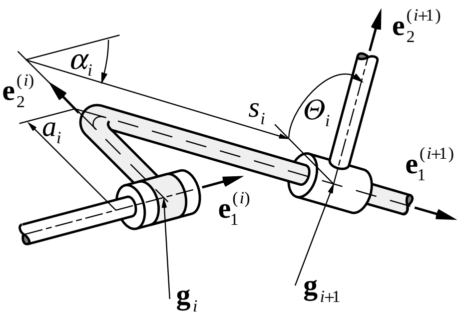


**Fig 3–14:** Cylindrical joint-body element.

- *Helical joint-body element*

The relative transformation between the coordinate systems $\underline{\mathbf{e}}^{(i)}$ and $\underline{\mathbf{e}}^{(i+1)}$ is in this case defined by the constant parameters $a_i$, $\alpha_i$ and the variable parameters $s_i$ and $\theta_i$. The parameters $s_i$ and $\theta_i$ are related by the equation $s_i = p_i * \theta_i$.

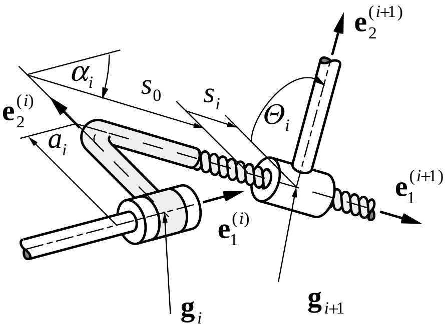


**Fig 3–15:** Helical joint-body element.